Stresses and Surcharge Stresses due to Plane Loading on Skewed Footings

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Introduction:
The distribution of stresses in earth masses is often estimated using the corresponding distribution in a linear elastic medium with boundary conditions approximating those in the problem of interest [7]. In some cases, elastic theory is also used to estimate displacements as well. Although soils do not behave as linear elastic materials, the rationale for this practice has been the availability of the solutions to problems for which the boundary conditions corresponded reasonably well to the boundary conditions for foundation engineering problems, as well as the lack of generally accepted alternatives. Experimental and analytical studies have been carried out to investigate the degree in which the results of elastic theory are applicable to earth masses. Perloff (1975)[11] and Harr (1977)[5] have summarized the conclusions of these investigations.

For the near edge or end of the footing area, it might be expected that certain amount of attenuation of stress with depth would occur, because no stress is applied beyond the edge. Similarly, with a loaded footing area of limited size, it might be expected that the applied stress at the surface would dissipate rather rapidly with depth.

For shallow foundation problems, solutions from the theory of elasticity or approximate methods are most commonly used to evaluate stresses with depth under areas of limited extent. Frequently used solutions from the theory of elasticity, usually in the form of convenient charts, can be found in Newmark (1942) [10], Fadum (1948) [2], Foster and Ahlvin (1954) [3], Scott (1963) [14], Harr (1966) [6], Poulos and Davis (1974) [13], Perloff (1975) [11], Perloff and Baron (1976) [12], Holtz and Kocacs (1981) [8], and U.S. Navy (1982) [21], among others. In some cases, the equations are given and solutions to often-used problems can be readily programmed on a microcomputer or programmable calculator, As these solutions are available in most design offices, they will not be presented here.

Point Load:
The most important original solution was given by Boussinesq (1885) [1] for the distribution of stresses within a linear elastic half-space resulting from a point load applied normal to the surface, illustrated in Fig. 1. The results obtained were

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{\left[r^2+z^2\right]^{\frac{5}{2}}} \qquad \text{.......... (1)}$$

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$$\sigma_r = \frac{P}{2\pi}\left[\frac{3zr^2}{\left[r^2+z^2\right]^{\frac{5}{2}}} - \frac{1-2\mu}{\sqrt{r^2+z^2}\left(\sqrt{r^2+z^2}+z\right)}\right] \quad \text{......} (2)$$

$$\sigma_r = \frac{P}{2\pi}(1-2\mu)\left[\frac{1}{\sqrt{r^2+z^2}\left(\sqrt{r^2+z^2}+z\right)} - \frac{z}{\left[r^2+z^2\right]^{\frac{3}{2}}}\right] \quad \text{......} (3)$$

$$\tau_{rz} = \frac{3P}{2\pi}\frac{z^2 r}{\left[r^2+z^2\right]^{\frac{5}{2}}} \quad \text{......} (4)$$

$$\tau_{\alpha z} = \tau_{r\alpha} = 0 \quad \text{......} (5)$$

Where μ is Poisson's ratio and other quantities in the equations are defined in Fig. 1. These stresses are the stresses which would occur in a weightless linear elastic medium. Preexisting stress due to the weight of the material must be superimposed upon these.

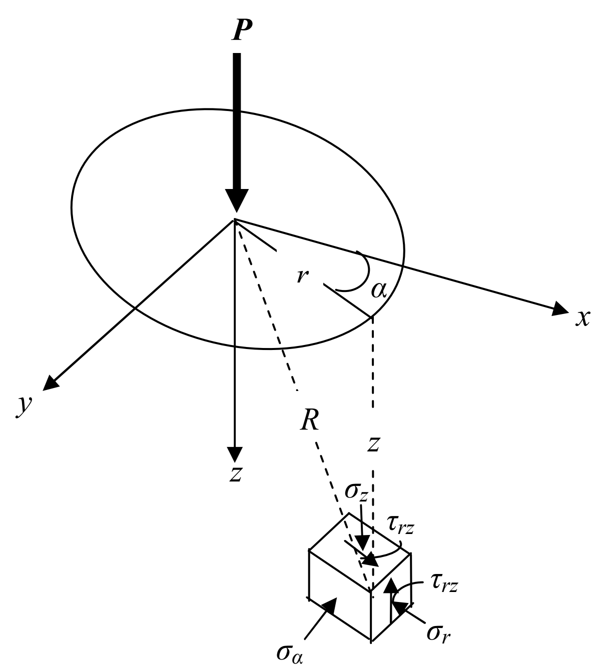

Fig. 1 Stresses in elastic half-space due to point load at the surface.

Rewriting the equation in terms of x and y using:

$\sigma_x = \sigma_r \cos^2\alpha + \sigma_\alpha \sin^2\alpha$, $\sigma_y = \sigma_r \sin^2\alpha + \sigma_\alpha \cos^2\alpha$, $\tau_{xy} = (\sigma_r - \sigma_\alpha)\sin\alpha\cos\alpha$

$\tau_{xz} = \tau_{rz}\cos\alpha$ and $\tau_{yz} = \tau_{rz}\sin\alpha$, gives two sets of equations:

Set #1

$$\sigma_x = \frac{3P}{2\pi}\frac{x^2 z}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (6)$$

$$\sigma_y = \frac{3P}{2\pi}\frac{y^2 z}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (7)$$

$$\tau_{xy} = \frac{3P}{2\pi}\frac{xyz}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (8)$$

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (9)$$

$$\tau_{xz} = \frac{3P}{2\pi}\frac{xz^2}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (10)$$

$$\tau_{yz} = \frac{3P}{2\pi}\frac{yz^2}{\left[x^2+y^2+z^2\right]^{\frac{5}{2}}} \quad \ldots\ldots (11)$$

Set #2

$$\sigma_x = \frac{(1-2\mu)P}{2\pi}\left[-\frac{x^2}{\left(x^2+y^2\right)^2} + \frac{zx^2}{\left(x^2+y^2\right)^2\sqrt{x^2+y^2+z^2}} + \frac{y^2}{\left(x^2+y^2\right)^2} - \frac{zy^2}{\left(x^2+y^2\right)^2\sqrt{x^2+y^2+z^2}} - \frac{zy^2}{\left(x^2+y^2\right)\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\right]$$

$$\ldots\ldots (12)$$

$$\sigma_y = \frac{(1-2\mu)P}{2\pi}\left[-\frac{y^2}{\left(x^2+y^2\right)^2} + \frac{zy^2}{\left(x^2+y^2\right)^2\sqrt{x^2+y^2+z^2}} + \frac{x^2}{\left(x^2+y^2\right)^2} - \frac{zx^2}{\left(x^2+y^2\right)^2\sqrt{x^2+y^2+z^2}} - \frac{zx^2}{\left(x^2+y^2\right)\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\right]$$

$$\ldots\ldots (13)$$

$$\tau_{xy} = \frac{(1-2\mu)P}{2\pi}\left[-\frac{2xy}{\left(x^2+y^2\right)^2} + \frac{2zxy}{\left(x^2+y^2\right)^2\sqrt{x^2+y^2+z^2}} + \frac{zxy}{\left(x^2+y^2\right)\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ (14)$$

The Boussinesq stresses becomes **<u>Set#1 +Set#2</u>**. Part of the reason of setting the solution in this manner, is when $\mu \approx .5$ the second set disappears. An example $\mu = .5$ for rubber.

Deriving the Stresses for Plane Loading on Footings:

To integrate Boussinesq equations using superimposition we have basically three types of loadings as in Fig. 2, 3, 4 and 5. The basic idea is to translate the point load equations in the x axis by $\bar{\xi}$ and in the y axis by $\bar{\lambda}$,as seen in the figures, then integrate with respect to $\bar{\xi}$ and $\bar{\lambda}$ over the footing $2a$ by $2b$ from $c–a$ to $c+a$ and $d–b$ to $d+b$ where the center of the footing is located at the coordinate $(c, d, 0)$.

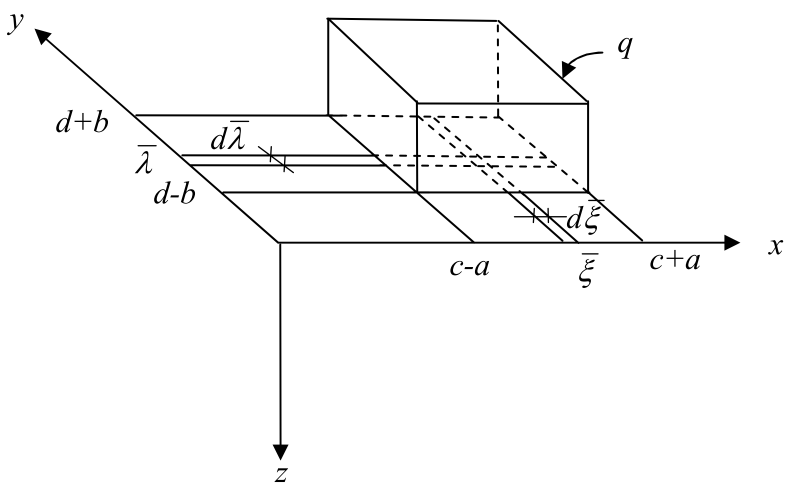

Fig. 2 Uniform Load q on 2b x 2a Footing

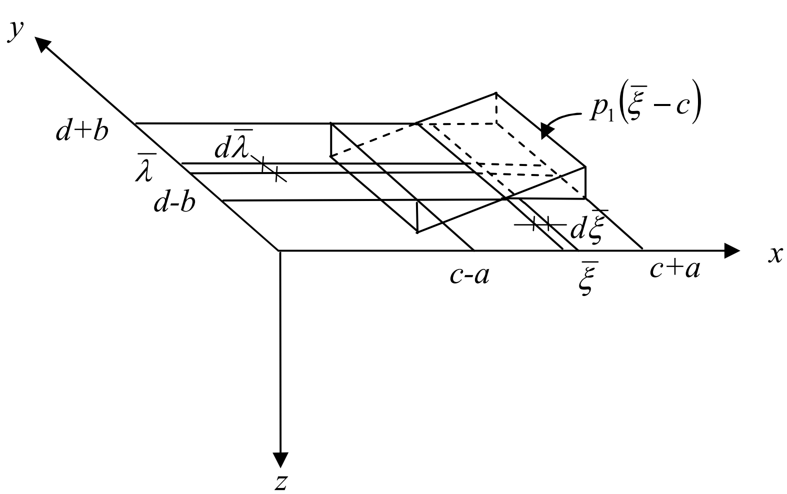

Fig. 3 Plane Load for p_1 in x on 2b x 2a Footing

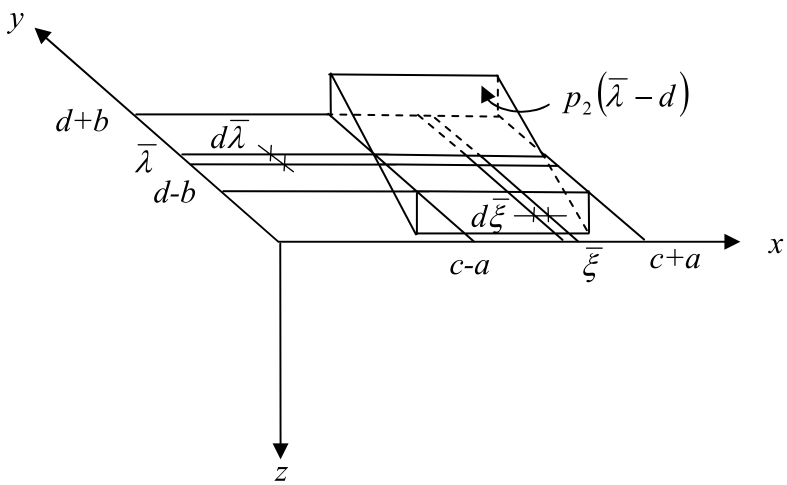

Fig. 4 Plane Load for p_2 in y on 2b x 2a Footing

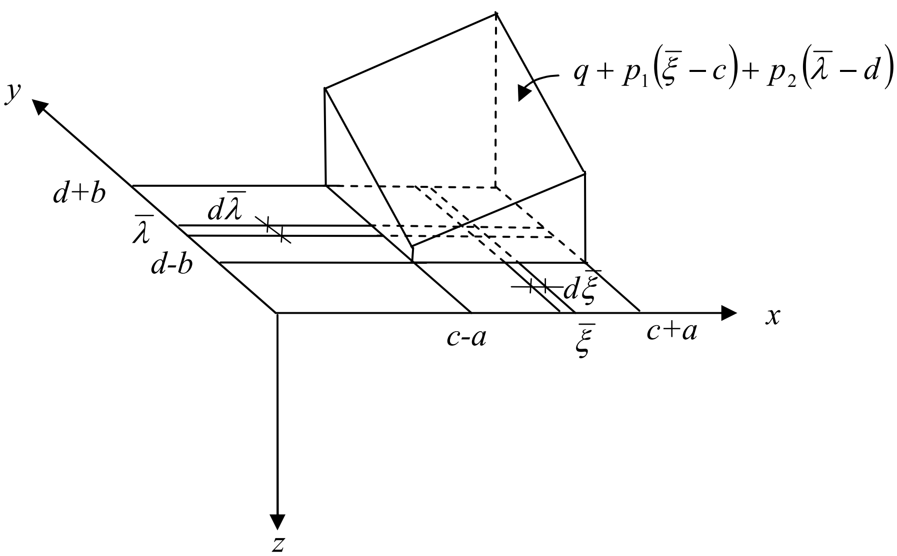

Fig. 5 Plane Load on 2b x 2a Footing

Thus we have three functions to integrate:

$$z_1 = q \qquad c - a \le \bar{\xi} \le c + a \quad \text{and} \quad d - b \le \bar{\lambda} \le d + b$$
$$z_2 = p_1(\bar{\xi} - c) \qquad c - a \le \bar{\xi} \le c + a \quad \text{and} \quad d - b \le \bar{\lambda} \le d + b$$
$$z_3 = p_1(\bar{\lambda} - c) \qquad c - a \le \bar{\xi} \le c + a \quad \text{and} \quad d - b \le \bar{\lambda} \le d + b \qquad \ldots\ldots (15)$$

And the three integrations over Boussinesq equations with respect to $\overline{\xi}$ and $\overline{\lambda}$ become:

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x-\overline{\xi}, y-\overline{\lambda}, z\right)\, q\, d\overline{\xi}\, d\overline{\lambda} \quad \text{(16)}$$

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x-\overline{\xi}, y-\overline{\lambda}, z\right)\, p_1\left(\overline{\xi}-c\right) d\overline{\xi}\, d\overline{\lambda} \quad \text{(17)}$$

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x-\overline{\xi}, y-\overline{\lambda}, z\right)\, p_2\left(\overline{\lambda}-d\right) d\overline{\xi}\, d\overline{\lambda} \quad \text{(18)}$$

Where σ_i is the Boussinesq stress equations 6 to 14 in *x*, *y* and *z* respectively and *i* represents the *x*, *y*, *z* stress components and directions. Also the *x* is replaced by $x-\overline{\xi}$ and the *y* is replaced by $y-\overline{\lambda}$ due to the translation of the axis. Now, translate the *x* axis back by -*c* and the *y* axis back by -*d* and the center of the footing becomes the origin of the axis. The equations become

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x+c-\overline{\xi}, y+d-\overline{\lambda}, z\right)\, q\, d\overline{\xi}\, d\overline{\lambda} \quad \text{(19)}$$

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x+c-\overline{\xi}, y+d-\overline{\lambda}, z\right)\, p_1\left(\overline{\xi}-\mathrm{c}\right) d\overline{\xi}\, d\overline{\lambda} \quad \text{(20)}$$

$$\int_{d-b}^{d+b} \int_{c-a}^{c+a} \sigma_i\left(x+c-\overline{\xi}, y+d-\overline{\lambda}, z\right)\, p_2\left(\overline{\lambda}-d\right) d\overline{\xi}\, d\overline{\lambda} \quad \text{(21)}$$

Now we change variable by letting $\overline{\xi} = c+\xi$ and $\overline{\lambda} = d+\lambda$ so $d\overline{\xi} = d\xi$ and $d\overline{\lambda} = d\lambda$ and the equations become:

$$\int_{-b}^{b} \int_{-a}^{a} \sigma_i\left(x-\xi, y-\lambda, z\right)\, q\, d\xi\, d\lambda \quad \text{(22)}$$

$$\int_{-b}^{b} \int_{-a}^{a} \sigma_i\left(x-\xi, y-\lambda, z\right)\, p_1 \xi\, d\xi\, d\lambda \quad \text{(23)}$$

$$\int_{-b}^{b} \int_{-a}^{a} \sigma_i\left(x-\xi, y-\lambda, z\right)\, p_2 \lambda\, d\xi\, d\lambda \quad \text{(24)}$$

Finally we let $u = x-\xi$ and $v = y-\lambda$ so $du = dx$ and $dv = dy$ and the equations become much simpler to integrate and to keep track, and they are:

$$\int_{y-b}^{y+b} \int_{x-a}^{x+a} \sigma_i\left(u, v\right)\, q\, du\, dv \quad \text{(25)}$$

$$\int_{y-b}^{y+b} \int_{x-a}^{x+a} \sigma_i\left(u, v\right)\, p_1\left(x-u\right) du\, dv \quad \text{(26)}$$

$$\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)\; p_2(y-v)\,du\,dv \quad \text{……………………………………………………………} (27)$$

Now combine all the constants together and separate from the line equation yields:

$$\sigma_{i1}=[q+p_1x+p_2y]\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)\,du\,dv \quad \text{………. for Set\#1………………………………} (28)$$

$$\sigma_{i2}=[q+p_1x+p_2y]\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)\,du\,dv \quad \text{………. for Set\#2………………………………} (29)$$

$$\sigma_{i3}=-p_1\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)u\,du\,dv \quad \text{…………………. for Set\#1…………………………} (30)$$

$$\sigma_{i4}=-p_2\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)v\,du\,dv \quad \text{………………… for Set\#1……………………………} (31)$$

$$\sigma_{i5}=-p_1\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)u\,du\,dv \quad \text{…………………… for Set\#2…………………………} (32)$$

$$\sigma_{i6}=-p_2\int_{y-b}^{y+b}\int_{x-a}^{x+a}\sigma_i(u,v)v\,du\,dv \quad \text{…………………… for Set\#2…………………………} (32)$$

The total stress becomes $\sigma_{it}=\sigma_{i1}+\sigma_{i2}+\sigma_{i3}+\sigma_{i4}+\sigma_{i5}+\sigma_{i6}$, where the following equations 33 to 60 has to be evaluated from $u=x+a$ to $u=x-a$ first then from $v=y+b$ to $v=y-b$ second the result becomes $\sigma_{it}=\sigma_{ik}(x+a,y+b,z)-\sigma_{ik}(x-a,y+b,z)-\sigma_{ik}(x+a,y-b,z)+\sigma_{ik}(x-a,y-b,z)$, and k = 1 to 6.

$$\sigma_{x1}=\frac{[q+xp_1+yp_2]}{2\pi}\left\{-\frac{uvz}{(u^2+z^2)\sqrt{u^2+v^2+z^2}}+\tan^{-1}\left[\frac{uv}{z\sqrt{u^2+v^2+z^2}}\right]\right\} \quad \text{………………..} (33)$$

$$\sigma_{y1}=\frac{[q+xp_1+yp_2]}{2\pi}\left\{-\frac{uvz}{(v^2+z^2)\sqrt{u^2+v^2+z^2}}+\tan^{-1}\left[\frac{uv}{z\sqrt{u^2+v^2+z^2}}\right]\right\} \quad \text{………………..} (34)$$

$$\tau_{xy1}=\frac{[q+xp_1+yp_2]}{2\pi}\left\{\frac{z}{\sqrt{u^2+v^2+z^2}}\right\} \quad \text{……………………………………………………} (35)$$

$$\sigma_{z1}=\frac{[q+xp_1+yp_2]}{2\pi}\left\{\frac{uvz}{\sqrt{u^2+v^2+z^2}}\left[\frac{1}{u^2+z^2}+\frac{1}{v^2+z^2}\right]+\tan^{-1}\left[\frac{uv}{z\sqrt{u^2+v^2+z^2}}\right]\right\} \quad \text{……….} (36)$$

$$\tau_{xz1}=\frac{[q+xp_1+yp_2]}{2\pi}\left\{-\frac{z^2v}{(u^2+z^2)\sqrt{u^2+v^2+z^2}}\right\} \quad \text{……………………………………………..} (37)$$

$$\tau_{yz1} = \frac{[q + xp_1 + yp_2]}{2\pi}\left\{-\frac{z^2 u}{(v^2 + z^2)\sqrt{u^2 + v^2 + z^2}}\right\} \quad \text{.. (38)}$$

$$\sigma_{x2} = \left(\frac{1-2\mu}{4\pi}\right)[q + xp_1 + yp_2]\left\{\tan^{-1}\left(\frac{v}{u}\right) - \tan^{-1}\left(\frac{u}{v}\right) + \tan^{-1}\left(\frac{u}{v}\frac{z}{\sqrt{u^2+v^2+z^2}}\right) - \tan^{-1}\left(\frac{v}{u}\frac{z}{\sqrt{u^2+v^2+z^2}}\right) - \tan^{-1}\left(\frac{1}{z}\frac{uv}{\sqrt{u^2+v^2+z^2}}\right)\right\}$$

.... (39)

$$\sigma_{y2} = \left(\frac{1-2\mu}{4\pi}\right)[q + xp_1 + yp_2]\left\{\tan^{-1}\left(\frac{u}{v}\right) - \tan^{-1}\left(\frac{v}{u}\right) + \tan^{-1}\left(\frac{v}{v}\frac{z}{\sqrt{u^2+v^2+z^2}}\right) - \tan^{-1}\left(\frac{u}{v}\frac{z}{\sqrt{u^2+v^2+z^2}}\right) - \tan^{-1}\left(\frac{1}{z}\frac{uv}{\sqrt{u^2+v^2+z^2}}\right)\right\}$$

............................…. (40)

$$\tau_{xy2} = \left(\frac{1-2\mu}{2\pi}\right)[q + xp_1 + yp_2]\left\{\frac{1}{4}\ln\left[\frac{u^2+v^2}{u^2+z^2}\right] + \frac{1}{4}\ln\left[\frac{u^2+v^2}{v^2+z^2}\right] + \ln\left[\frac{z+\sqrt{u^2+v^2+z^2}}{\sqrt{u^2+v^2}}\right]\right\} \quad \text{......... (41)}$$

$$\sigma_{x3} = \frac{p_1}{2\pi}\left[2z\ln\left(\frac{v+\sqrt{u^2+v^2+z^2}}{z}\right) + \frac{vu^2 z}{(u^2+z^2)\sqrt{u^2+v^2+z^2}}\right] \quad \text{.. (42)}$$

$$\sigma_{y3} = \frac{p_1}{2\pi}\left[z\ln\left(\frac{v+\sqrt{u^2+v^2+z^2}}{z}\right) - \frac{vz}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{... (44)}$$

$$\tau_{xy3} = \frac{p_1}{2\pi}\left[z\ln\left(\frac{u+\sqrt{u^2+v^2+z^2}}{z}\right) - \frac{uz}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{... (45)}$$

$$\sigma_{z3} = \frac{p_1}{2\pi}\left[\frac{vz^3}{(u^2+z^2)\sqrt{u^2+v^2+z^2}}\right] \quad \text{.. (46)}$$

$$\tau_{xz3} = \frac{p_1}{2\pi}\left\{\frac{vz^2}{\left(u+\sqrt{u^2+v^2+z^2}\right)\sqrt{u^2+v^2+z^2}} + \frac{uvz^2}{\sqrt{u^2+v^2+z^2}}\left[\frac{1}{u^2+z^2} + \frac{1}{v^2+z^2}\right] - z\tan^{-1}\left[\frac{uv}{z\sqrt{u^2+v^2+z^2}}\right]\right\}$$

.... (47)

$$\tau_{yz3} = \frac{p_1}{2\pi}\left[-\frac{z^2}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{.. (48)}$$

$$\sigma_{x4} = \frac{p_2}{2\pi}\left[z\ln\left(\frac{u+\sqrt{u^2+v^2+z^2}}{z}\right) - \frac{uz}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{... (49)}$$

$$\sigma_{y4} = \frac{p_2}{2\pi}\left[2z\ln\left(\frac{u+\sqrt{u^2+v^2+z^2}}{z}\right)+\frac{uv^2z}{\left(v^2+z^2\right)\sqrt{u^2+v^2+z^2}}\right] \quad \text{(50)}$$

$$\tau_{xy4} = \frac{p_2}{2\pi}\left[z\ln\left(\frac{v+\sqrt{u^2+v^2+z^2}}{z}\right)-\frac{vz}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{(51)}$$

$$\sigma_{z4} = \frac{p_2}{2\pi}\left[\frac{uz^3}{\left(v^2+z^2\right)\sqrt{u^2+v^2+z^2}}\right] \quad \text{(52)}$$

$$\tau_{xz4} = \frac{p_2}{2\pi}\left[-\frac{z^2}{\sqrt{u^2+v^2+z^2}}\right] \quad \text{(53)}$$

$$\tau_{yz4} = \frac{p_2}{2\pi}\left\{\frac{uz^2}{\left(v+\sqrt{u^2+v^2+z^2}\right)\sqrt{u^2+v^2+z^2}}+\frac{uvz^2}{\sqrt{u^2+v^2+z^2}}\left[\frac{1}{u^2+z^2}+\frac{1}{v^2+z^2}\right]-z\tan^{-1}\left[\frac{uv}{z\sqrt{u^2+v^2+z^2}}\right]\right\} \quad \text{(54)}$$

$$\sigma_{x5} = \left(\frac{1-2\mu}{2\pi}p_1\right)\left\{v+v\ln\left[\frac{z+\sqrt{u^2+v^2+z^2}}{|v|}\right]-z\ln\left[\frac{\sqrt{u^2+z^2}}{|z|}\right]\right\} \quad \text{(55)}$$

$$\sigma_{y5} = \left(\frac{1-2\mu}{2\pi}p_1\right)\left\{-v-v\ln\left[\frac{z+\sqrt{u^2+v^2+z^2}}{|v|}\right]-z\ln\left[\frac{v+\sqrt{u^2+v^2+z^2}}{\sqrt{u^2+z^2}}\right]\right\} \quad \text{(56)}$$

$$\tau_{xy5} = \left(\frac{1-2\mu}{2\pi}p_1\right)\left\{-u+v\tan^{-1}\left(\frac{u}{v}\right)+z\ln\left[\frac{u+\sqrt{u^2+v^2+z^2}}{|z|}\right]-v\tan^{-1}\left(\frac{u}{v}\frac{z}{\sqrt{u^2+v^2+z^2}}\right)\right\} \quad \text{(57)}$$

$$\sigma_{x6} = \left(\frac{1-2\mu}{2\pi}p_2\right)\left\{-u-u\ln\left[\frac{z+\sqrt{u^2+v^2+z^2}}{|u|}\right]-z\ln\left[\frac{u+\sqrt{u^2+v^2+z^2}}{\sqrt{v^2+z^2}}\right]\right\} \quad \text{(58)}$$

$$\sigma_{y6} = \left(\frac{1-2\mu}{2\pi}p_2\right)\left\{u+u\ln\left[\frac{z+\sqrt{u^2+v^2+z^2}}{|u|}\right]-z\ln\left[\frac{\sqrt{v^2+z^2}}{|z|}\right]\right\} \quad \text{(59)}$$

$$\tau_{xy6} = \left(\frac{1-2\mu}{2\pi}p_2\right)\left\{-v+u\tan^{-1}\left(\frac{v}{u}\right)+z\ln\left[\frac{v+\sqrt{u^2+v^2+z^2}}{|z|}\right]-u\tan^{-1}\left(\frac{v}{u}\frac{z}{\sqrt{u^2+v^2+z^2}}\right)\right\}$$

………………………………. (60)

Because of the amount of integrations was immense setting up a verification spread sheet was needed. (***verify.xls***). The solution was verified using numerical differentiations on the solution to (equation 33 through 60) and the result is checked to mach Boussinesq equations 6 through 14. The result is summarized in the spread sheet ***stress.xls*** for the stresses.

Surcharge Stresses for Plane Loading on Skewed Footings:

Re-examining the Boussinesq equations for horizontal stress due to concentrated load on the surface as shown in Fig. 6 and given in equations 6 through 14. However, if AB represents the boundary between soil and wall, the deformation along this line, Δ, is usually much smaller than attained in the interior of soil mass. Mindlin (1936) [9] has shown that consequently would be about twice the amount computed by equations 6 through 14. This derivation is usually done using mirror image loading

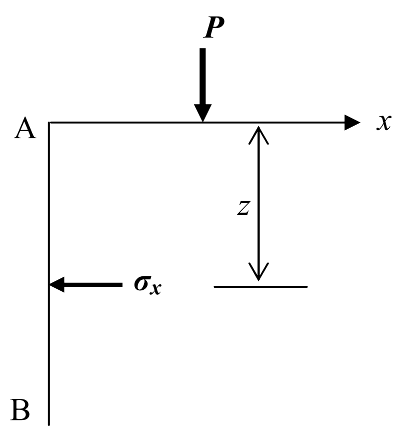

Fig. 6 Earth pressure due to point load.

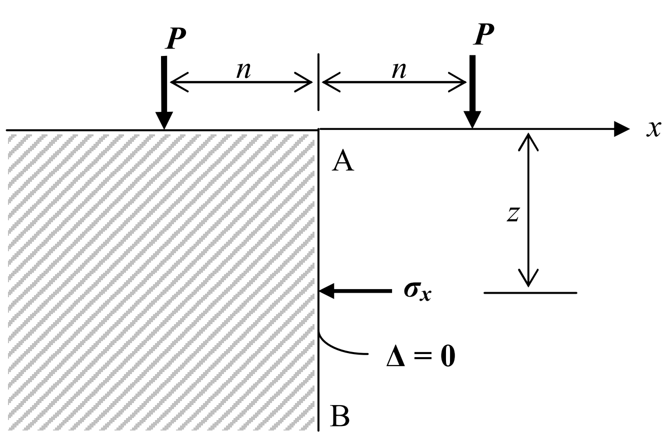

Fig. 7 Earth pressure on wall due to point load using mirror images.

analysis as in Fig. 7. By removing the shaded area in the left side on Fig. 7 the solution is obtained as twice the loading with $\Delta = 0$.

Spangler (1938) [16] and Gerber (1929) [4] verified experimentally Mindlin mirror image solution. More discussion can be found in Terzaghi & Peck (1967) [18] and Spangler & Handy (1973) [15] books. Recommendations by the Sheet Piling Design Manual [17], Terzaghi (1954) [19], Holl (1940) [13] and Spangler & Handy (1973) [15] has been using Boussinesq equations with $\mu = 0.5$. This assumption of allowing no tension in the material, will be shown can be unsafe for footings. It may be ok in some cases to ignore the Set#2 equations 12, 13 and 14 in matching certain experiments. However, it is a definite misleading assumption when it comes to generalizing for all types of soils and conditions. In the case of Teng (1962) [20] for a strip load and in the case of line loads, Set#2 disappears in the integrations for σ_x without having to have $\mu = 0.5$ and their solution is correct. Wu (1975) [22] propose to use Boussinesq equations as is without setting $\mu = 0.5$. It is known that

$$\mu = \frac{k_0}{k_0 + 1} \quad \text{......} \quad (61)$$

where k_0 is the at rest coefficient and there is no reason not to use a value for μ and investigate the difference.

For the general solution of a skewed footing by an angle θ from the x axis as in Fig. 8 it is as follows:

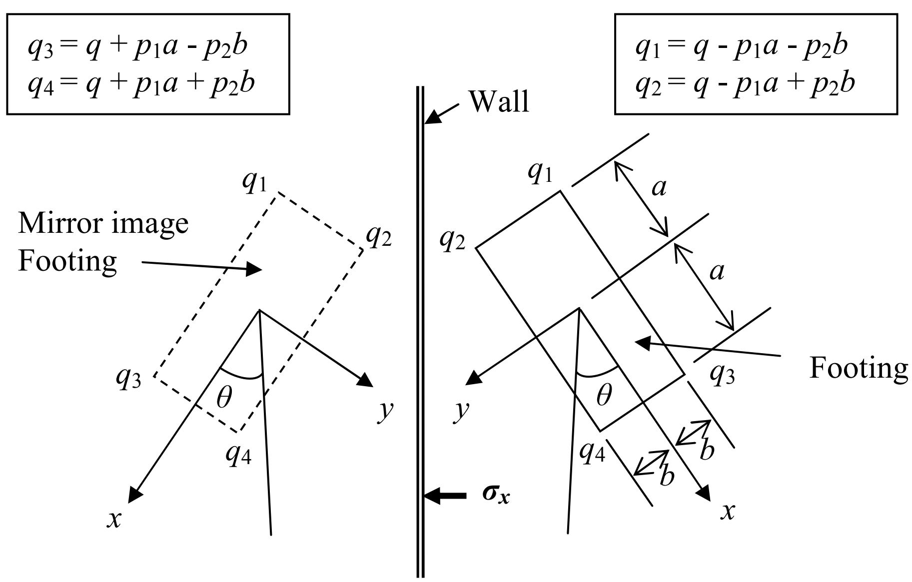

Fig. 8 Earth pressure on wall due to footing load using mirror images

$$\sigma = \frac{(\sigma_{xt} + \sigma_{yt})}{2} + \frac{(\sigma_{xt} - \sigma_{yt})}{2}\cos 2\theta - \tau_{xyt}\sin 2\theta \quad \text{(62)}$$

$$\tau = -\tau_{xyt}\cos 2\theta + \frac{(\sigma_{xt} - \sigma_{yt})}{2}\sin 2\theta \quad \text{(63)}$$

$$\sigma_x\big|_{wall} = 2\sigma \quad \text{(64)}$$

See ***surcharge.xls*** file for the surcharge stresses. To show the comparison when using $\mu = 0.5$, a real life problem will be used. The project was a shoring for a tunnel used as an exit on the I90 freeway. The exit name is I90 to Mercer Island tunnel. Above the tunnel there is an overpass. It is the Island Crest Way Bridge with a skewed footing making $\theta = 51.5$ degrees from the wall see photo FIG 11. The footing size is 127.72 ft by 19 ft, $\mu = 0.28$, $q = 5.45$ ksf, $p_1 = -0.5$ kcf, $p_2 = 0$, we examine the stress in the z direction first for the coordinate $x = -70.86$ ft and $y = 3$ ft. Fig 9 shows the comparison. We find in this case when $\mu = 0.5$ the maximum stress was 1.595 ksf and when $\mu = 0.28$ the maximum stress was 1.219 ksf it overestimated the maximum stress by 31%.

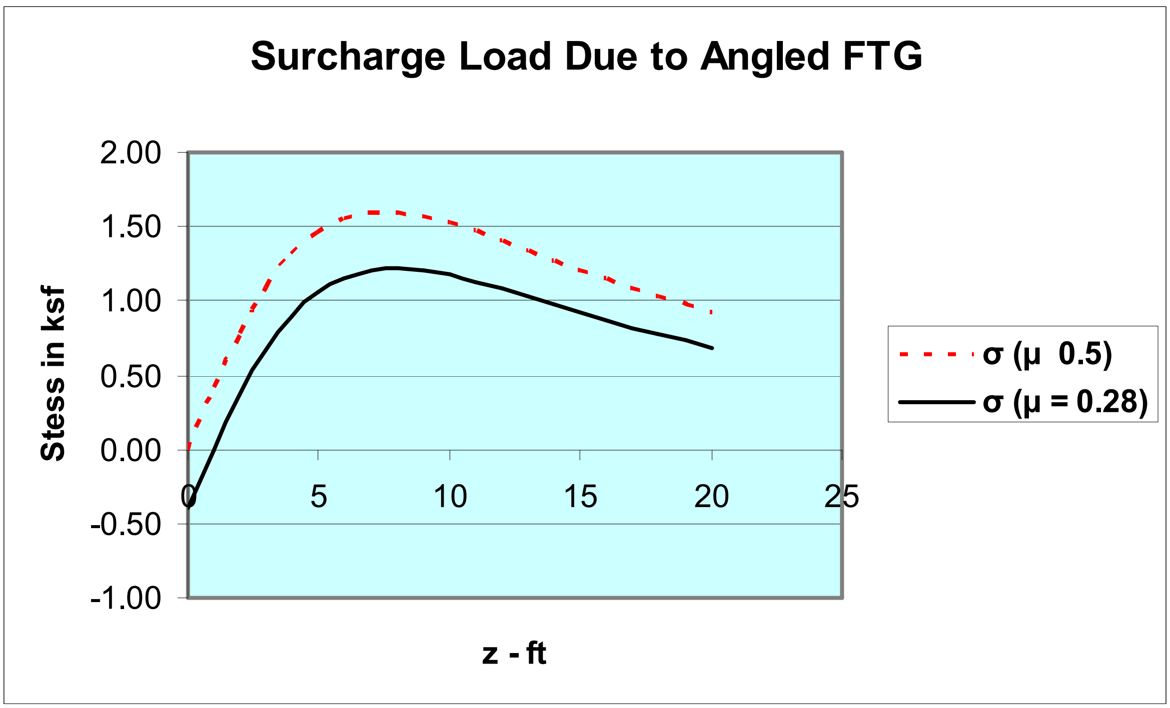

Fig. 9 - I90 to Mercer Island Shoring Wall at Tunnel (**x = -70.86 ft, y = 3 ft, θ = 51.5°**)

Now we examine the stress in the z direction for the coordinate $x = -78.86$ ft and $y = -4$ ft. Fig 10 shows the comparison. We find in this case when $\mu = 0.5$ the maximum stress was 0.496 ksf and when $\mu = 0.28$ the maximum stress was .487 ksf it overestimated the maximum stress by 2% and also the stress at $z = 0$ is .226 ksf. In this situation the stresses on the wall are higher at the top adding a considerable moment on the wall. Hopefully this example sways engineers to investigate the proper μ and not ignore the Set#2 of the equations. Now it is true that when using the Set#2 of the equations one may have tension in the soil which it is normally set to zero if it is not of too great. In any case whether the designer choose $\mu = 0.5$ so no tension is allowed or use the actual μ, all of the compressive pressures must be taken into account and only common sense can be used for setting small tension stresses to zero, when it is close to zero. Ultimately, the designer must make the proper decision using the proper equations. For safety, it should be clear when using the above equations for footings the second set#2 must be taken into account.

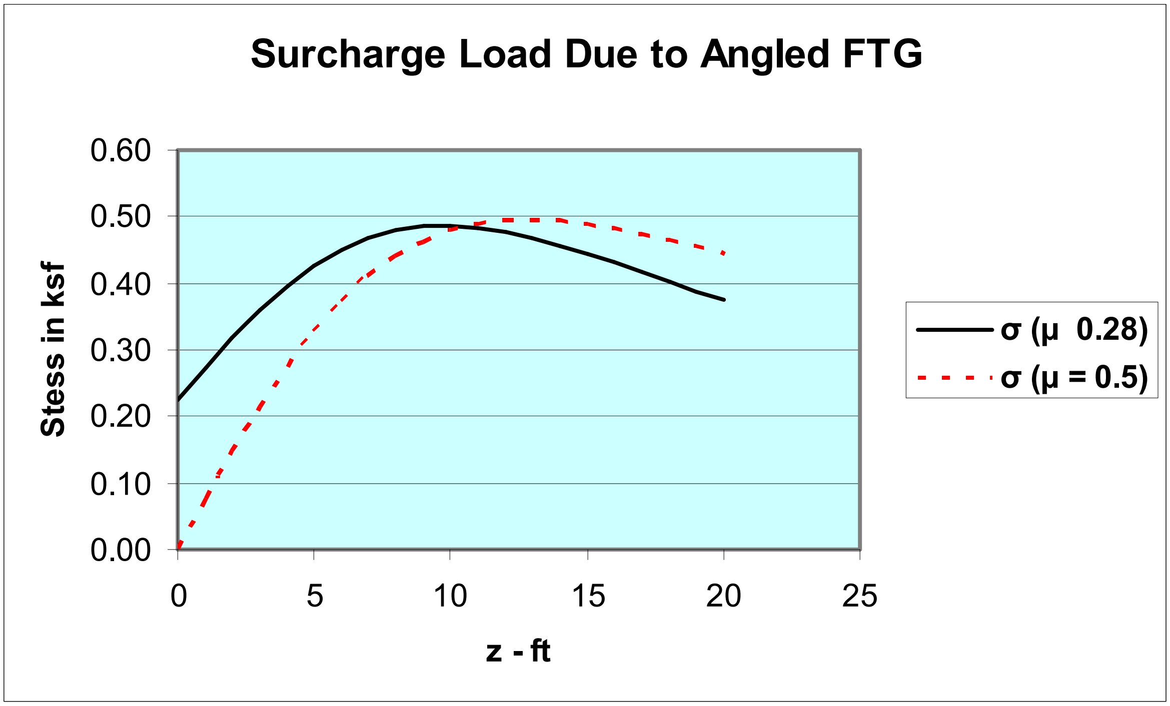

Fig. 10 - I90 to Mercer Island Shoring Wall at Tunnel (***x* = -78.86 ft, *y* = -4 ft, θ = 51.5°**)

Fig 11 - Island Crest Way Tunnel Exit from I90 East from Express Freeway

Conclusion

The solution for stresses due to a footing that has a plane loading equation are found by integrating Boussinesq equations at any point in the media. The solution is also extended to find the surcharge stresses due of a skewed footing on a wall. The result shows that ignoring the material properties can be unsafe and the designer must examine both conditions in the above equations when the values of the Poisson’s ration 0.5 and the actual.

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