Slope Stability Slip Surface Using Variational Methods

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Abstract:
In this synopsis, the Variational method is used to determine the slope stability slip surface based on the ordinary method of slices without pore pressure and not circular for the time being. It is shown that using the ordinary method of slices gives approximately the same shape slip surface for the moment's equation, so ignoring that the method was derived for circles is adequate. The result shows that the embankment's geometry and topography affect the slip surface's shape.

Introduction:
The principle of slope stability has been developed over the past seventy years and provides a set of soil mechanics principles from which to approach practical problems. Although the mechanics of slope failure in heap leaching may be difficult to predict, the principles used in a standard of practice examination are relatively straightforward. The proposed method of variation analysis is a far better prediction and is a refined method than current methods; the slip surface is prescribed and not guest at. This approach relieves the mathematical uncertainty of what the slip surface is, provided the soil parameters are close to reality. It gave us a better prediction than a circle or log spiral.

An analysis of slope stability begins with the hypothesis that a slope's stability results from downward or motivating forces (i.e., gravitational) and resisting (or upward) forces. These forces act in equal and opposite directions, as seen in practice. The resisting forces must be greater than the motivating forces for a slope to be stable. The relative stability of a slope (or how stable it is at any given time) is typically conveyed by geotechnical engineers through a Factor of Safety F_s defined as follows:

$$F_s = \frac{\sum R}{\sum M}$$

The equation states that the factor of safety is the ratio between the forces/moments resisting (R) movements and the forces/moments motivating (M) movements. When the safety factor equals 1.0, a slope has just reached failure conditions. If the safety factor falls below 1.0, failure or has already occurred is imminent. Factors of safety in the range of 1.3 to 1.5 are considered reasonably safe in many design scenarios. However, the actual factor of safety used in the design is influenced by the risk involved and the certainty with which other variables are known.

Analysis 1:

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Minimizing the safety factor for the ordinary method of slices would give a closer slip surface for a true slip surface. Once the slip surface is prescribed, a comprehensive framework for limit equilibrium methods of slices developed, for example, by Fredlund et al. (1981), would give a more realistic safety factor. The Fredlund methodology can analyze both circular and non-circular slip surfaces. Because pore pressure can change the slip surface, it will not be considered for the demonstration. So, the forgoing analysis is for one condition, which will be checked for a unique situation with φ is zero or for a cohesion material. Still, the slip surface derived by Chouery in determining the maximum soil pressure for a smooth wall, currently being published, should be considered since the Cullman method is always considered. In this case, the slip surface by Chouery assumes a smooth wall, which is appropriate for slope stability of the embankment where there is no friction at the outer surface. Also, the safety factor of the forces and the moments must be considered separately; both have to be minimized, and the least one must be considered.

a) Force Analysis

The ordinary method of slices gives the first safety factor of the forces as follows:

$$F = \frac{c\int ds + \gamma \tan\phi \int y\cos\alpha dx}{\gamma\int y\sin\alpha dx} = \frac{\int F_1(x, y, \dot{y})dx}{\int F_2(x, y, \dot{y})dx} = \frac{\int F_1}{\int F_2} \quad \text{.......} (1)$$

Where $\dot{y} = -\tan\alpha$

Now the solution must satisfy the following:

$$F = \frac{Min(\int F_1)}{Max(\int F_2)} \quad \text{.......} (2)$$

Rewrite Eq. 2 as

$$FMax(\int F_2) - Min(\int F_1) = 0 \quad \text{.......} (3)$$

Since $\int F_1$ is a positive number, then Eq. 3 can be written as

$$FMax(\int F_2) + Max(-\int F_1) = 0 \quad \text{.......} (4)$$

Or

$$Max(F\int F_2 - \int F_1) = 0 \quad \text{.......} (5)$$

If F is picked, Eq 5 can be satisfied using the Variational method. The constant coefficients of the nonlinear differential equation will satisfy Eq. 5. Now, pick a lower number for F than before and solve Eq. 5 again and continue until Eq. 5 cannot be satisfied anymore. The solution is found for minimum F in Eq. 2. This situation is similar to minimizing Eq. 2 using a Lagrange multiplier, so seeking to minimize $\int F_1$ with the condition $\int F_2$ is a constant or vice versa. Thus, the extremum that gives the slip surface is

$$Max(\int F_2 + \lambda \int F_1) \quad \ldots\ldots (6)$$

Where λ is the Lagrange multiplier, thus if $\lambda = 0$, it is Max($\int F_2$); if $\lambda = -\infty$ then it is Min($\int F_1$), and λ is not necessarily $1/F$, λ balance the maximization.

b) <u>Moment Analysis</u>

See Fig 1 for a stationary slip surface a-b. For a moment at point A, where A is to be determined, the equation for the safety factor is:

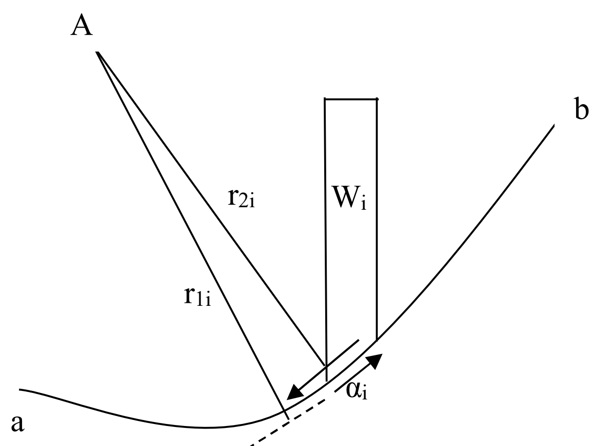

Fig 1 – Slice i taking moments at A

$$F_m = \frac{c\sum r_{1i}\Delta l_i + \tan\phi \sum r_{1i}\cos\alpha_i W_i}{\sum r_{2i}\sin\alpha_i W_i} \quad \ldots\ldots (7)$$

Or

$$F_m = \frac{r_1 \int F_1}{r_2 \int F_2} \quad \ldots\ldots (8)$$

Where r_1 and r_2 are the resultant radial distances, with many respect $r_{1i} \approx r_{2i}$. Thus

$$F_m = \frac{Min(r_1 \int F_1)}{Max(r_2 \int F_2)} = \frac{Min(r_1) \times Min(\int F_1)}{Max(r_2) \times Max(\int F_2)} \quad \text{(9)}$$

Min($\int F_1$) is two parts the $\int ds$ and $\int y\cos\alpha\, dx$. The minimum $\int ds$ is a line for minimum cohesion; thus, it forces the slip surface to be less concave upward, and r_1 is reduced to a minimum, $\int y\cos\alpha\, dx$ is minimum when W_i (or y) is reduced, giving α_i to increase and cosα_i is to be decreased thus again r_1 is reduced to a minimum since the shape is less concave upward. Thus Min($\int F_1$) gives r_1 at a minimum anyway.

Max($\int F_2$) is the maximum when $\int y\sin\alpha\, dx$ increase or W_i (or y) increase decreases α_i. Thus, r_2 is increased for some constant α_i because the slip surface becomes more concave upward. If we kept W_i (or y) constant and moved α_i up or down by a small increment, then r_{2i} increases for some slices and decreases for others, and r_2 remains relatively the same. Thus, the bulk of the maximization of r_2 is increasing W_i and not increasing sinα_i. Also, if we increase concavity after Max($\int F_2$) is reached, $\int y\sin\alpha\, dx$ will decrease, and r_2 will increase but remain relatively the same, and the product $r_2 \int F_2$ may decrease since $\int F_2$ decrease. Thus Max($\int F_2$) gives r_2 at the approximately maximum anyway. In reality, Eq. 9 will provide a different slip surface than Eq. 2. However, from the Ordinary Method of Slices (Fellenius, 1927) and Bishop's Modified Method (Bishop, 1955)

- Only for circular slip surfaces.
- Satisfies moment equilibrium.
- Satisfies vertical force equilibrium.
- Does not satisfy horizontal force equilibrium.

It is expected the difference between F_m (Moment Analysis), and F (Force Analysis) is negligible.

This assumption of concavity upwardly is based on physics and experiments since if the slip surface is concave downward for a given c and φ is not possible. Thus Eq. 9 becomes:

$$F_m \approx \frac{r_1 \times Min(\int F_1)}{r_2 \times Max(\int F_2)} \quad \text{(10)}$$

It reduces to maximizing Eq. 6. Thus, the slip surface for Eq. 2 and Eq. 10 has the same shape; only the constant coefficients from solving the nonlinear differential equation of Eq. 6 are different and make sense physically. Thus, the only slip surfaces to be investigated are from Eq. 6.

Eq. 6 can be written as

$$-\gamma\int_{x_1}^{x_2}\frac{\dot{y}[y+f(x)]}{\sqrt{1+\dot{y}^2}}dx+\lambda\left[c\int_{x_1}^{x_2}\sqrt{1+\dot{y}^2}\,dx+\gamma\tan\phi\int_{x_1}^{x_2}\frac{y+f(x)}{\sqrt{1+\dot{y}^2}}dx\right] \quad \ldots\ldots\ldots\ldots (11)$$

Where $f(x)$ is the function representing the topography, we apply the Euler equation.

$$\frac{\partial\Re}{\partial y}-\frac{d}{dx}\left[\frac{\partial\Re}{\partial\dot{y}}\right]=0 \quad \text{and} \quad \Re=-\gamma\frac{\dot{y}(y+f(x))}{\sqrt{1+\dot{y}^2}}+\lambda\left[c\sqrt{1+\dot{y}^2}+\gamma\tan\phi\frac{y+f(x)}{\sqrt{1+\dot{y}^2}}\right] \quad \ldots\ldots\ldots\ldots (12)$$

As before with Chouery's article, we start with a Fourier series representation of $f(x)$ and convert it to a Taylor series by inverting the matrix representation and solving Eq. 12 with a polynomial. $y=\sum_{n=0}^{m}a_n x^n$ Where m gives a good approximation of the curve where the coefficient a_{m+1} diminishes if substituting x by x/y_0, where y_0 is the maximum height of the embankment.

We conclude that $f(x)$ affects the slip surface, geometry, and topography, but we do not know how much. Also, the slip surface is prescribed, not guessed at. Examples will follow.

Examples 1:

The solution for $f(x) = 0$ and $\varphi = 0$ will be sought. Note: the ordinary method of slices gives the same answer as other methods. Fig 2 shows the embankment for $\lambda = 0$.

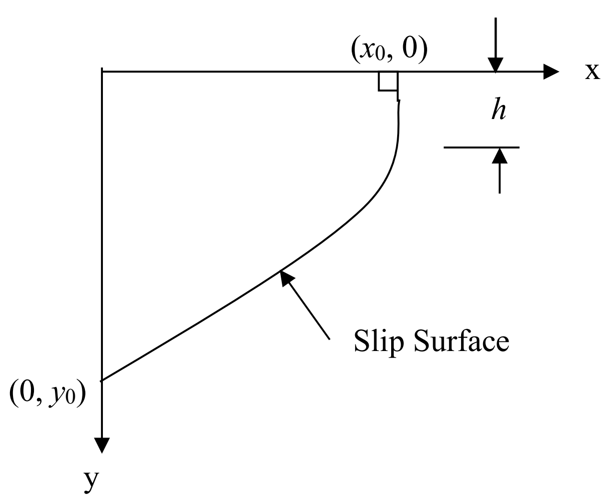

Fig 2 Slip Surface for $f(x) = 0$, $\varphi = 0$ and $\lambda = 0$

From Eq. 12 yields,

$$\Re = -\gamma \frac{\dot{y}y}{\sqrt{1+\dot{y}^2}} + \lambda\left[c\sqrt{1+\dot{y}^2}\right] \quad \text{and} \quad \Re - \dot{y}\frac{\partial \Re}{\partial \dot{y}} = h \quad \text{.............................. (13)}$$

If set $\lambda = 0$, the solution yields:

$$\dot{y} = \pm \frac{h^{\frac{1}{3}}}{\sqrt{y^{\frac{2}{3}} - h^{\frac{2}{3}}}} \quad \text{and} \quad \dot{x} = \pm h^{-\frac{1}{3}}\sqrt{y^{\frac{2}{3}} - h^{\frac{2}{3}}} \quad \text{.. (14)}$$

Where h is a constant, and the slip surface becomes:

$$x = \pm h^{-\frac{1}{3}} \int \sqrt{y^{\frac{2}{3}} - h^{\frac{2}{3}}}\, dy + k \quad \text{... (15)}$$

Where k is a constant of integrations. The factor of safety is:

$F = \dfrac{3.845c}{\gamma y_0}$ and $h = 0.1733\, y_0$, x_0 (at $y = h$) is $x_0 = 0.866\, y_0$, $\dot{y}\big|_{y=y_0} = -0.6716$ or 33.885^{o}

Where the vertical portion at h in Fig. 2 is taken as having a cohesion c. Note that a circle gives 3.83 instead of 3.845; thus, $\lambda \neq 0$ will be calculated in the next examples. Also: note the slip surface does not look like a circle or log spiral and is 90 degrees to the top surface. F_m exact is not calculated in this exercise.

Example 2:

When considering $\lambda \neq 0$ in example 1 and using the Euler equation of Eq. 12, it yields:

$$-\frac{\dot{y}}{\sqrt{1+\dot{y}^2}} - \frac{d}{dx}\left[-\frac{y}{\sqrt{1+\dot{y}^2}} + \frac{y\dot{y}^2}{\left(1+\dot{y}^2\right)^{\frac{2}{3}}} + \lambda\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}\right] = 0 \quad \text{............................... (16)}$$

Where in here λ is a new constant. Eq. 16 yields:

$$\ddot{y} = -\frac{\dot{y}^3 + \dot{y}^5}{3y\dot{y} - \lambda(1+\dot{y}^2)} \quad \text{... (17)}$$

Working with a fourth-order Taylor polynomial approximation series, so let

$$\frac{y}{y_0} = a_0 + a_1\left(\frac{x}{y_0}\right) + a_2\left(\frac{x}{y_0}\right)^2 + a_3\left(\frac{x}{y_0}\right)^3 + a_4\left(\frac{x}{y_0}\right)^4 \quad \text{....................................... (18)}$$

$$\frac{\dot{y}}{y_0} = \frac{a_1}{y_0} + \frac{2a_2}{y_0}\left(\frac{x}{y_0}\right) + \frac{3a_3}{y_0}\left(\frac{x}{y_0}\right)^2 + \frac{4a_4}{y_0}\left(\frac{x}{y_0}\right)^3$$

or …………………………………….. (19)

$$\dot{y} = a_1 + 2a_2\left(\frac{x}{y_0}\right) + 3a_3\left(\frac{x}{y_0}\right)^2 + 4a_4\left(\frac{x}{y_0}\right)^3$$

$$\ddot{y} = \frac{2a_2}{y_0} + \frac{6a_3}{y_0}\left(\frac{x}{y_0}\right) + \frac{12a_4}{y_0}\left(\frac{x}{y_0}\right)^2 \quad \text{…………………………………………………… (20)}$$

$$\dddot{y} = \frac{6a_3}{y_0^2} + \frac{24a_4}{y_0^2}\left(\frac{x}{y_0}\right) \quad \text{………………………………………………………………… (21)}$$

Now let

$$g\left(\frac{x}{y_0}\right) = -\frac{\dot{y}^3 + \dot{y}^5}{3y\ddot{y} - \lambda(1+\dot{y}^2)} \quad \text{………………………………………………………… (22)}$$

$g(x/y_0)$ can be expressed with a Taylor approximation as

$$g\left(\frac{x}{y_0}\right) = g(0) + \frac{1}{1!}g'(0)\left(\frac{x}{y_0}\right) + \frac{1}{2!}g''(0)\left(\frac{x}{y_0}\right)^2 \quad \text{…………………..……………………… (23)}$$

From Eq. 17, 20 and 22

$$\frac{2a_2}{y_0} = g(0) = -\frac{a_1^3 + a_1^5}{3y_0a_1 - \lambda(1+a_1^2)}$$

or ……………………………………………………….... (24)

$$a_2 = \frac{y_0}{2}\left[-\frac{a_1^3 + a_1^5}{3y_0a_1 - \lambda(1+a_1^2)}\right]$$

$$\frac{6a_3}{y_0} = \frac{1}{1!}g'(0) = \frac{3\dot{y}^5 + 3\dot{y}^7 - 6y(\dot{y}^3 + 2\dot{y}^5)\ddot{y} + 3\lambda(\dot{y}^2 + 2\dot{y}^4 + \dot{y}^6)\ddot{y}}{\left[3y\dot{y} - \lambda(1+\dot{y}^2)\right]^2}$$

or

$$a_3 = \frac{y_0}{6}\left\{\frac{3a_1^5 + 3a_1^7 - 12(a_1^3 + 2a_1^5)a_2 + 6\lambda(a_1^2 + 2a_1^4 + a_1^6)\frac{a_2}{y_0}}{\left[3y_0 a_1 - \lambda(1+a_1^2)\right]^2}\right\}$$

…………………………………………………………. (25)

$$\frac{12a_4}{y_0} = \frac{1}{2!}g''(0) = \frac{\begin{array}{c}15\dot{y}^4\ddot{y} + 21\dot{y}^6\ddot{y} - 6\dot{y}(\dot{y}^3 + 2\dot{y}^5)\ddot{y} - 6y(3\dot{y}^2 + 10\dot{y}^4)\ddot{y}^2 - 6y(\dot{y}^3 + 2\dot{y}^5)\dddot{y} \\ + 3\lambda(2\dot{y} + 8\dot{y}^3 + 6\dot{y}^5)\ddot{y}^2 + 3\lambda(\dot{y}^2 + 2\dot{y}^4 + \dot{y}^6)\dddot{y}\end{array}}{\left[3y\dot{y} - \lambda(1+\dot{y}^2)\right]^2} - \frac{2\left[3\dot{y}^5 + 3\dot{y}^7 - 6y(\dot{y}^3 + 2\dot{y}^5)\ddot{y} + 3\lambda(\dot{y}^2 + 2\dot{y}^4 + \dot{y}^6)\ddot{y}\right]\left[3\dot{y}^2 + 3y\ddot{y} - 2\lambda\dot{y}\ddot{y}\right]}{\left[3y\dot{y} - \lambda(1+\dot{y}^2)\right]^3}$$

or

$$a_4 = \frac{y_0}{24}\left\{\frac{\begin{array}{l}(30a_1^4 + 42a_1^6)\frac{a_2}{y_0} - 12a_1(a_1^3 + a_1^5)\frac{a_2}{y_0} - 24y_0(3a_1^2 + 5a_1^4)\left(\frac{a_2}{y_0}\right)^2 - 36(a_1^3 + a_1^5)\frac{a_3}{y_0} \\ + 12\lambda(2a_1 + 8a_1^3 + 6a_1^5)\left(\frac{a_2}{y_0}\right)^2 + 18\lambda(a_1^2 + 2a_1^4 + a_1^6)\frac{a_3}{y_0^2}\end{array}}{\left[3y_0 a_1 - \lambda(1+a_1^2)\right]^2} - \frac{2\left[3a_1^5 + 3a_1^7 - 12(a_1^3 + 2a_1^5)a_2 + 6\lambda(a_1^2 + 2a_1^4 + a_1^6)\frac{a_2}{y_0}\right]\left[3a_1^2 + 6a_2 - 4\lambda a_1\frac{a_2}{y_0}\right]}{\left[3y_0 a_1 - \lambda(1+a_1^2)\right]^3}\right\}$$

………………………………………………………………….(26)

Thus, pick a_1 and λ given y_0, find a_2 from Eq. 24, substitute in Eq. 25, find a_3 and substitute in Eq. 26 and find a_4. Set $y = 0$ in Eq. 18, find the proper root x0, and minimize F in Eq. 1 using Eq. 18 and 19.

$$F = \frac{c\int_0^{x_0} \sqrt{1+\dot{y}^2}\,dx}{-\gamma\int_0^{x_0} \frac{\dot{y}y}{\sqrt{1+\dot{y}^2}}\,dx} \quad \text{...} (27)$$

To compare with example 1, the solution for a fourth-degree polynomial for $\lambda = 0$ is:

$y_0 = 20$

$a_1 = -0.85$ (on bottom 40.36°) in example 1 it was -0.6716 (33.885°)

$a_2 = -0.207$

$a_3 = -7.728 \times 10^{-3}$

$a_4 = -3.469 \times 10^{-4}$

x_0 (at $y = 0$) = 0.949 y_0 in example 1 it was 0.866 y_0 (at surface tanα = 1.289 or 52.19°)

$$F = \frac{3.898c}{\gamma y_0}$$

So, the result shows the fourth-order polynomial approximation is 1.5% higher than example 1. In order to be closer to F of example 1, more terms are needed in the Taylor series polynomial. All numbers are close and not obtained from an algorithm, so there could be computation or derivation errors. The effect of λ has a solution as follows:

$y_0 = 20$

$\lambda = -19.55$

$a_1 = -0.683$ (on bottom 34.33°)

$a_2 = -0.38$

$a_3 = -0.027$

$a_4 = -8.035 \times 10^{-4}$

x_0 (at $y = 0$) = 0.94 y_0 (at surface tanα = 1.526 or 56.76°)

$$F = \frac{3.855c}{\gamma y_0}$$

Or F is lower for using λ but not quite as example 1 because more terms are needed in the polynomial.

<u>Example 3</u>:

We seek a different solution than Example 2. Thus, considering $\lambda \neq 0$ in example 1 and using the Euler equation of Eq. 12, it yields:

$$-\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}-\frac{d}{dx}\left[-\frac{y}{\sqrt{1+\dot{y}^2}}+\frac{y\dot{y}^2}{\left(1+\dot{y}^2\right)^{\frac{2}{3}}}+\lambda\frac{\dot{y}}{\sqrt{1+\dot{y}^2}}\right]=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(28)}$$

Where in here λ is a new constant. Eq. 28 yields:

$$\ddot{x}=\frac{\dot{x}^2+1}{3y\dot{x}-\lambda(\dot{x}^2+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(29)}$$

Working with a fourth-order Taylor polynomial approximation series, so let

$$\frac{x}{y_0}=a_0+a_1\left(\frac{y}{y_0}\right)+a_2\left(\frac{y}{y_0}\right)^2+a_3\left(\frac{y}{y_0}\right)^3+a_4\left(\frac{y}{y_0}\right)^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

$$\frac{\dot{x}}{y_0}=\frac{a_1}{y_0}+\frac{2a_2}{y_0}\left(\frac{y}{y_0}\right)+\frac{3a_3}{y_0}\left(\frac{y}{y_0}\right)^2+\frac{4a_4}{y_0}\left(\frac{y}{y_0}\right)^3$$

or $\ldots\ldots\ldots\ldots\ldots\ldots$ (31)

$$\dot{x}=a_1+2a_2\left(\frac{y}{y_0}\right)+3a_3\left(\frac{y}{y_0}\right)^2+4a_4\left(\frac{y}{y_0}\right)^3$$

$$\ddot{x}=\frac{2a_2}{y_0}+\frac{6a_3}{y_0}\left(\frac{y}{y_0}\right)+\frac{12a_4}{y_0}\left(\frac{y}{y_0}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(32)}$$

$$\dddot{y}=\frac{6a_3}{y_0^2}+\frac{24a_4}{y_0^2}\left(\frac{y}{y_0}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(33)}$$

Now let

$$g\left(\frac{y}{y_0}\right)=-\frac{\dot{y}^3+\dot{y}^5}{3y\dot{y}-\lambda(1+\dot{y}^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(22)}$$

$g(y/y_0)$ can be expressed with a Taylor approximation as

$$g\left(\frac{y}{y_0}\right) = g(0) + \frac{1}{1!}g'(0)\left(\frac{y}{y_0}\right) + \frac{1}{2!}g''(0)\left(\frac{y}{y_0}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(35)}$$

From Eq. 29, 32 and 34

$$\frac{2a_2}{y_0} = g(0) = -\frac{1}{\lambda}$$
$$or \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(36)}$$
$$a_2 = -\frac{y_0}{2\lambda}$$

$$\frac{6a_3}{y_0} = \frac{1}{1!}g'(0) = \frac{9y\dot{x}^2\ddot{x} - 3\dot{x}(1+\dot{x}^2) + 3y\ddot{x}}{\left[3y\dot{x} - \lambda(1+\dot{x}^2)\right]^2}$$
$$or$$
$$a_3 = -\frac{y_0}{2}\left[\frac{a_1}{\lambda^2(1+a_1^2)}\right]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(37)}$$

$$\frac{12a_4}{y_0} = \frac{1}{2!}g''(0) = \frac{18y\dot{x}\ddot{x}^2 + 3y\dddot{x}}{\left[3y\dot{x} - \lambda(1+\dot{x}^2)\right]^2} - \frac{2\left[9y\dot{x}^2\ddot{x} - 3\dot{x}(1+\dot{x}^2) + 3y\ddot{x}\right]\left[3\dot{x} - 3y\ddot{x} - 2\lambda\dot{x}\ddot{x}\right]}{\left[3y\dot{x} - \lambda(1+\dot{x}^2)\right]^3}$$
$$or$$
$$a_4 = -\frac{5y_0}{4\lambda^3}\frac{a_1}{(1+a_1^2)^2}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(38)}$$

Thus pick a_1 and λ gave y_0, find a_2 from Eq. 36, find a_3 in Eq. 37, and find a_4 in Eq. 38. Set $\dot{x} = 0$ in Eq. 31 and find the proper root y and make sure it is negative and minimize F in Eq. 1 using Eq. 31.

$$F = \frac{c\int_0^{y_0}\sqrt{1+\dot{x}^2}\,dy}{-\gamma\int_0^{y_0}\frac{\dot{x}y}{\sqrt{1+\dot{x}^2}}\,dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(39)}$$

The effect of λ has a solution as follows:

$y_0 = 20$

$\lambda = 17.5$

$a_1 = -0.35$ (at surface tan$\alpha = 2.857$ or 70.71° this is closer to 90° similar to example 1)

$a_2 = -0.571$

$a_3 = 0.01$

$a_4 = -4.535 \times 10^{-4}$

$a_0 = - a_1 - a_2 - a_3 - a_4 = 0.912$ @ $y = y_0$ and $x = 0$ in Eq. 30, or $x_0 = 0.912\, y_0$
(it was $x_0 = 0.938\, y_0$ in example 2 by solving the fourth-order polynomial of Eq. 18 with $x = x_0$ and $y = 0$.

$\dot{x}\big|_{y=y_0} = a_1 + 2a_2 + 3a_3 + 4a_4 = -1.464$ or $\alpha_0 = 34.335°$ (at bottom $\alpha_0 = 34.33°$ in Example 2)

$$F = \frac{3.808c}{\gamma y_0}$$

Thus, this polynomial approximation gave a factor for F of 3.808 less than a circle of an F factor of 3.83, and the Variational method works.

Note: λ takes a positive value due to the following:

$$\frac{1}{-\lambda}\int y \sin\alpha dx = \frac{1}{-\lambda}\int \frac{y}{\sqrt{1+\dot{x}^2}} dx = \frac{1}{\lambda}\int \frac{y\dot{x}}{\sqrt{1+\dot{x}^2}} dy = -\frac{1}{-\lambda}\int \frac{y\dot{y}}{\sqrt{1+\dot{y}^2}} dx$$

Conclusion:

Geometry affects the shape of the slip surface, but we do not know how much. Also, from the example shown, one may conclude that maximizing Max($\int F_2$), independent from c and φ, is all that is needed and adds a safety factor. It is not correct; the example shows otherwise. Also, generalization per one condition is not acceptable for one condition, and when including φ, the difference between a circle and a log spiral can be 13%. For example, the slip surface can be circular on the bottom and linear at the top, so what is the correct slip surface? Thus, deriving the correct slip surface is more desirable since the shape affects the safety factor. Note: finite element relies on Poisson's Ratio, and the Modulus of Elasticity has not produced a significant difference over the limit equilibrium method. The slip surface runs through the elements, is poorly defined, and has no plasticity. The slip surface is somewhere in the elements depending on the element's size. The finite element is still an approximation where Variational Method represents the exact slip surface given the Taylor series higher terms diminish. There is expected to be a difference with the Variational method and produce more reliable safety factors. If we say the hill is safe, we had better make sure.

Other issues: In many cases providing soil parameters is impossible to obtain. For example:
1- Embankments underwater in a deep ocean environment
2- Quik clay materials have no values.
3- A crater in a far-away planet or moon in our solar system.
4- Generally, boring logs are impossible to drill because of government or public restrictions.
5- No knowledge of soil below

This situation can be handled as in Example 1, c has to be estimated because if φ and c are zero, it would be like water.

Photos showing slip surface from Soil Mechanics, SI Version book by T. William Lambe and Robert V. Whitman, Massachusetts Institute of Technology, With the assistance of H. G. Poulos, University of Sydney, John Wiley & Sons, Inc. SI Version Copyright © 1979, pp 164 and pp 190.

It is a slope stability failure curve or a stress failure curve. It looks like it is similar to my slope stability curve. Per Colomb stress failure curve, it would be a line. Failure curves in the following photos are misunderstood.

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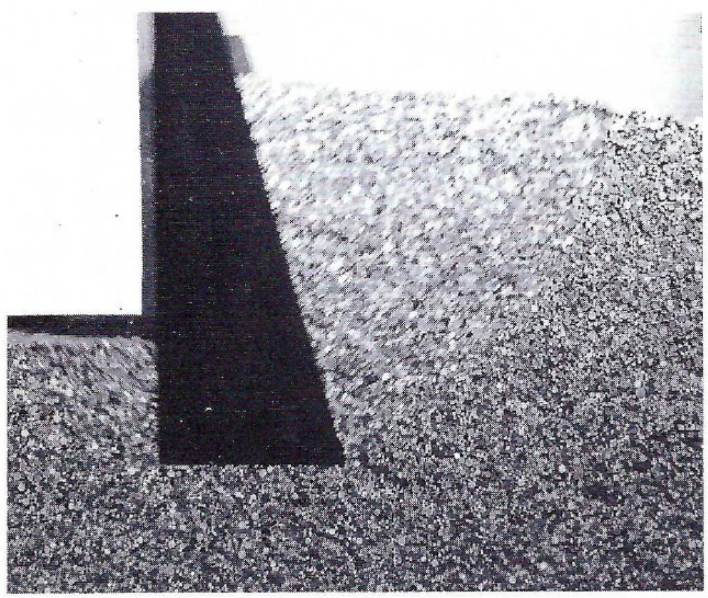

Fig. 13.3 Double exposure showing movements of "soil" surrounding model retaining wall.

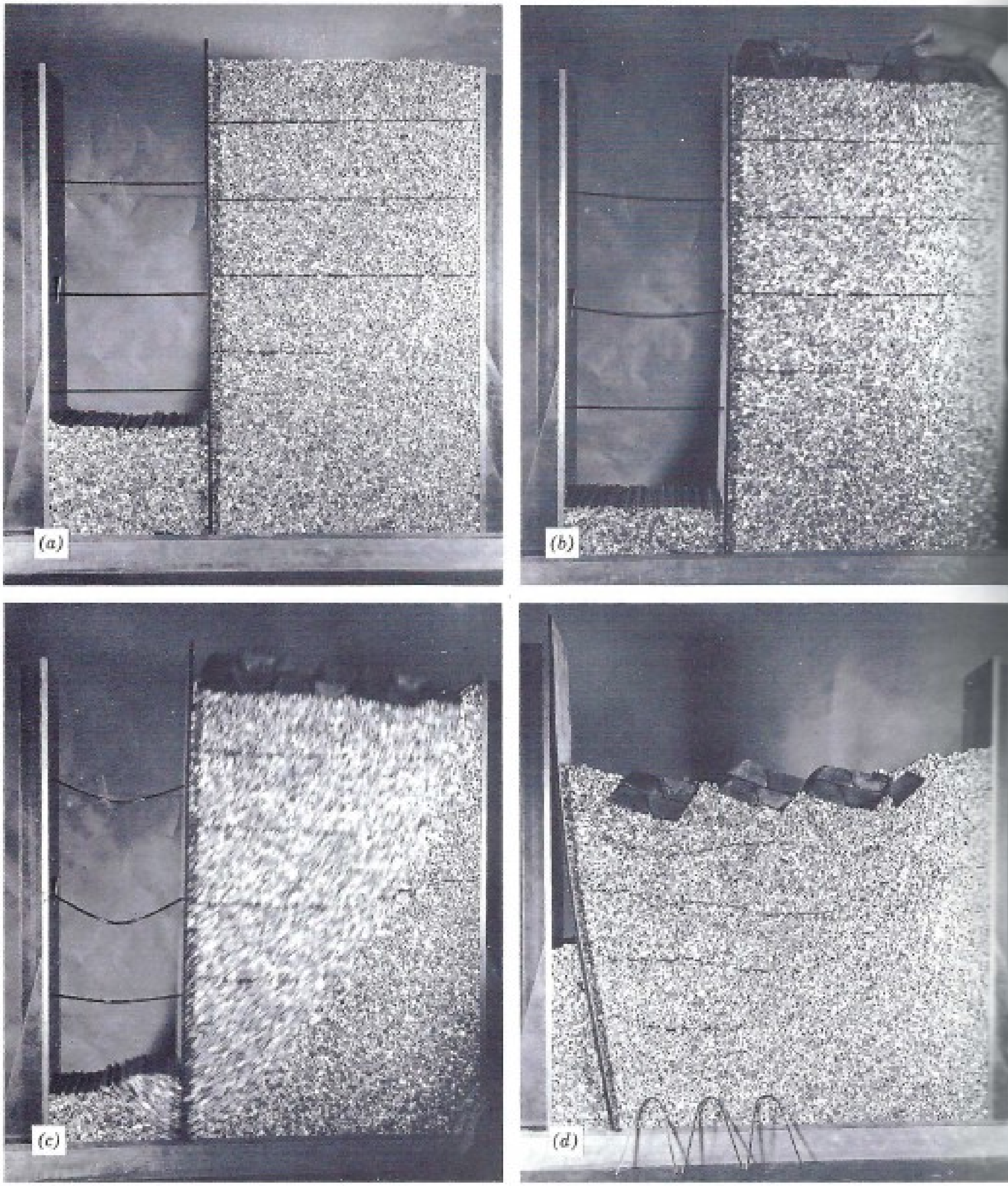

Fig. 13.30 Failure of model of braced excavation. (*a*) Stable. (*b*) About to fail. (*c*) Failing; note motions. (*d*) After failure.