Slope Stability Slip Surface Using Variational Methods
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Analysis 2:

We address the slope stability under heavy rainfall. Our reference is "Comparison study between traditional and finite element methods for slopes under heavy rainfall" by M. Rabie - Civil Engineering Dept., He/wan University, Cairo, Egypt
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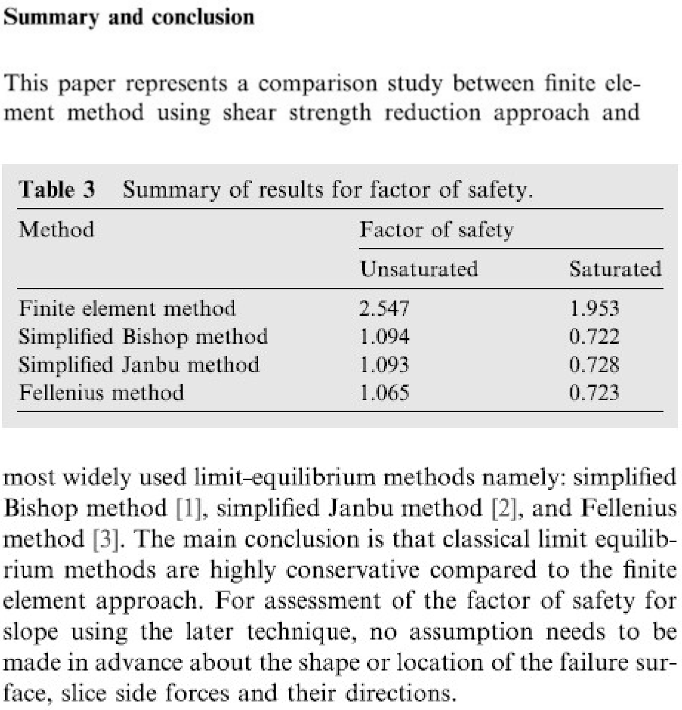
M. Rabie's summary shows the Finite Element



Summary and conclusion

This paper represents a comparison study between finite element method using shear strength reduction approach and

Table 3 Summary of results for factor of safety.

Method	Factor of safety	
	Unsaturated	Saturated
Finite element method	2.547	1.953
Simplified Bishop method	1.094	0.722
Simplified Janbu method	1.093	0.728
Fellenius method	1.065	0.723

most widely used limit-equilibrium methods namely: simplified Bishop method [1], simplified Janbu method [2], and Fellenius method [3]. The main conclusion is that classical limit equilibrium methods are highly conservative compared to the finite element approach. For assessment of the factor of safety for slope using the later technique, no assumption needs to be made in advance about the shape or location of the failure surface, slice side forces and their directions.

These numbers make me nervous, and I cannot accept Finite Element Analysis that way. From Fellenius 1927 to Duncan 1996, everything went well for 70 years. Now we say it is conservative. The finite element relies on Poisson's Ratio and the Modulus of Elasticity; they are elastic parameters; the soil is not elastic and cannot take tension. Trying to use Finite Elements for everything is a scientific fanatism.

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From Reference Army Corps of Engineers Manual EM 1110-2-1902

C-12. Use of the Finite Element Method

a. General. The finite element method (FEM) can be used to compute displacements and stresses caused by applied loads. However, it does not provide a value for the overall factor of safety without additional processing of the computed stresses. The principal uses of the finite element method for design are as follows:

(1) Finite element analyses can provide estimates of displacements and construction pore water pressures. These may be useful for field control of construction, or when there is concern for damage to adjacent structures. If the displacements and pore water pressures measured in the field differ greatly from those computed, the reason for the difference should be investigated.

(2) Finite element analyses provide displacement pattern which may show potential and possibly complex failure mechanisms. The validity of the factor of safety obtained from limit equilibrium analyses depends on locating the most critical potential slip surfaces. In complex conditions, it is often difficult to anticipate failure modes, particularly if reinforcement or structural members such as geotextiles, concrete retaining walls, or sheet piles are included. Once a potential failure mechanism is recognized, the factor of safety against a shear failure developing by that mode can be computed using conventional limit equilibrium procedures.

(3) Finite element analyses provide estimates of mobilized stresses and forces. The finite element method may be particularly useful in judging what strengths should be used when materials have very dissimilar stress-strain and strength properties, i.e., where strain compatibility is an issue. The FEM can help identify local regions where "overstress" may occur and cause cracking in brittle and strain softening materials. Also, the FEM is helpful in identifying how reinforcement will respond in embankments. Finite element analyses may be useful in areas where new types of reinforcement are being used or reinforcement is being used in ways different from the ways for which experience exists. An important input to the stability analyses for reinforced slopes is the force in the reinforcement. The FEM can provide useful guidance for establishing the force that will be used.

b. Use of finite element analyses to compute factors of safety. If desired, factors of safety equivalent to those computed using limit equilibrium analyses can be computed from results of finite element analyses. The procedure for using the FEM to compute factors of safety are as follows:

(1) Perform an analysis using the FEM to determine the stresses for the slope.

(2) Select a trial slip surface.

(3) Subdivide the slip surface into segments.

(4) Compute the normal stresses and shear stresses along an assumed slip surface. This requires interpolation of values of stress from the values calculated at Gauss points in the finite element mesh to obtain values at selected points on the slip surface. If an effective stress analysis is being performed, subtract pore pressures to determine the effective normal stresses on the slip surface. The pore pressures are determined from the same finite element analysis if a coupled analysis was performed to compute stresses and deformations. The pore pressures are determined from a separate steady seepage analysis if an uncoupled analysis was performed to compute stresses and deformations.

c. Advantages and disadvantages. Where estimates of movements as well as factor of safety are required to achieve design objectives, the effort required to perform finite element analyses can be justified. However, finite element analyses require considerably more time and effort, beyond that required for limit equilibrium analyses and additional data related to stress-strain behavior of materials. Therefore, the use of finite element analyses is not justified for the sole purpose of calculating factors of safety.

M. Rabie solution:

Simplified Bishop Method

This method satisfies vertical force equilibrium for each slice and overall moment equilibrium about the center of the circular trial surface. The simplified Bishop method also assumes zero interslice shear forces.

Thus, Bishop's method could be used to compute a factor of safety (FOS) = F for noncircular surfaces, where FOS is factor of safety = "F" and can be calculated as follows:-

$$F = \frac{\sum_{i=1}^{n}(C + N' \tan\phi)}{\sum_{i=1}^{n} A_5 - \sum_{i=1}^{n} A_6 + \sum_{i=1}^{n} A_7} \tag{1}$$

where:

$$A_5 = (W(1 - k_v) + U_\beta \cos\beta + Q\cos\delta)\sin\alpha \tag{2}$$

$$A_6 = (U_\beta \sin\beta + Q\sin\delta)\left(\cos\alpha - \frac{h}{R}\right) \tag{3}$$

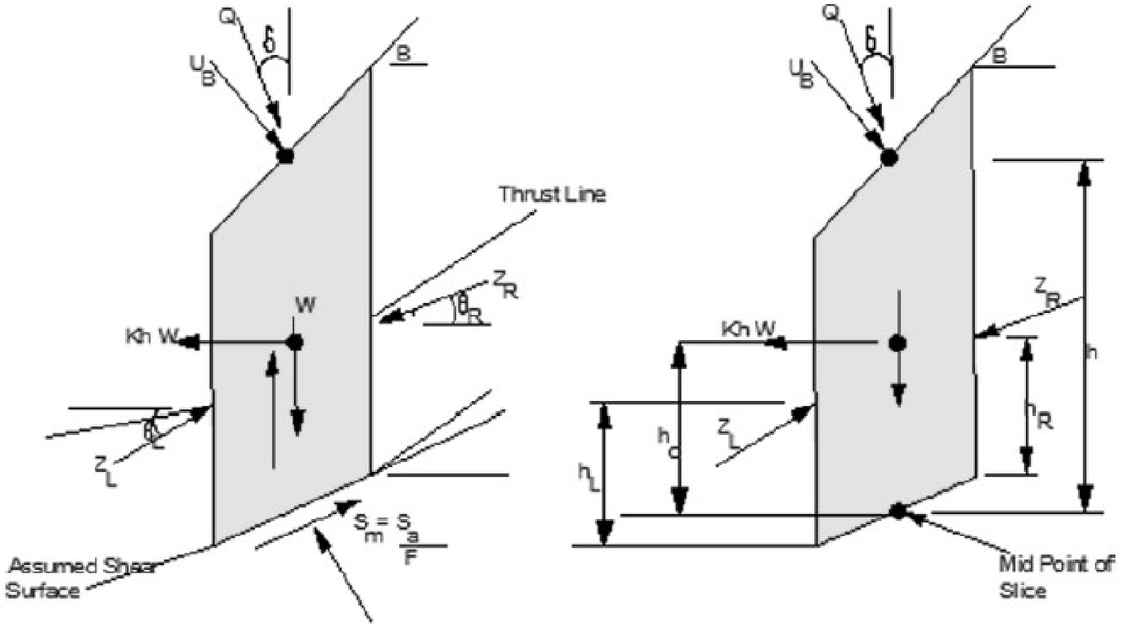
$$A_7 = k_h W\left(\cos\alpha - \frac{h_c}{R}\right) \tag{4}$$

R = the resistance force

We will not use the Janbu method because the forces balance themselves as the slice width becomes zero. Now we use *f(x)* for the topography above *y*, and *H is* for water height above zero; also, we are not using a circle, and β and δ are zero, so the equations become:

$$F = \frac{c\int_{x_1}^{x_2}\sqrt{1+\dot{y}^2}\,dx + \tan\varphi\left[\int_{x_1}^{x_2}\frac{\gamma\left(y+f(x)\right)+Q+\gamma_w(H-f(x))}{\sqrt{1+\dot{y}^2}}dx - U_\alpha\int_{x_1}^{x_2}\frac{dx}{\sqrt{1+\dot{y}^2}}\right]}{\int_{x_1}^{x_2}\frac{\gamma(1-k_v)\left(y+f(x)\right)+Q+\gamma_w(H-f(x))}{\sqrt{1+\dot{y}^2}}\dot{y}dx + k_h\int_{x_1}^{x_2}\frac{\gamma\left(y+f(x)\right)}{\sqrt{1+\dot{y}^2}}dx}$$

Reference Army Corps of Engineers Manual EM 1110-2-1902, equations C12 and C14.



F = factor of safety
S_a = available strength
= $C + N' \tan\phi$
S_m = mobilized strength
U_α = pore water force
U_β = surface water force
W = weight of slice
N' = effective normal force
Q = external surcharge
k_v = vertical seismic coefficient
k_h = horiz. seismic coefficient

Z_L = left interslice force
Z_R = right interslice force
θ_L = left interslice force angle
θ_R = right interslice force angle
h_L = height to force Z_L
h_R = height to force Z_R
α = inclination of slice base
β = inclination of slice top
δ = inclination of surcharge
b = width of slice
h = average height of slice
h_c = height to centroid of slice

Fig. 5 Forces acting on a typical slice.

M. Rabie Results:

Analysis of results

Figs. 11 and 12 show the results of total displacement increments for un-saturated and saturated slope obtained from finite element analysis, respectively. It is obvious that displacement increments for un-saturated slope are much higher than the saturated one. Besides, Figs. 13 and 14 illustrate the results of safety factors for un-saturated and saturated slope obtained from finite element analysis, respectively. The un-saturated slope gives safety factor of 2.547, while the saturated slope gives lower safety factor of 1.953.

The results of safety factors obtained from limit equilibrium (traditional) methods are listed in Table 3. It can be seen that the

limit equilibrium methods namely: simplified Bishop Method [1], simplified Janbu method [2], and Fellenius method [3] give low value for the safety factors for saturated and unsaturated slopes. On the other hand, finite element method gives high safety factors for saturated and unsaturated slopes compared to the methods which are based on limit equilibrium concept.

Moreover, there is no wide variation in the factors of safety calculated using classical limit equilibrium methods because they are assessed based on the same concept.

However, there is a wide discrepancy between the conventional and finite element methods in assessment safety factors, and this may be attributed to one of the following reasons:

Classical limit equilibrium methods depend on the directions of the forces acting on each slice in the slope are assumed;

In finite element approach, the factor of safety emerges naturally from analysis without the user having to commit to any particular form of mechanism a priori;

Limit equilibrium methods require a continuous surface passing the soil mass. This surface is essential in calculating

the minimum factor of safety (FOS) against sliding or shear failure.